Nova Scotia Examinations

Mathematics 12

Web Sample 2

Student Booklet

General Instructions - WEB SAMPLE*

This examination is composed of two sections with the following suggested time allotment:

Selected-Response (Multiple-Choice) Questions   Value 25 pts   (approx. 40 min)*
Constructed-Response Questions   Value 75 pts   (approx. 120 min)

*note: there are 35 constructed response questions on the Math NSEs as of January 2008

Total time: 3 hours (revision time included)

Use these suggested times to guide you in the completion of the examination; however, you might not find it necessary to spend the suggested time on each section. Plan your time to enable you to complete the examination.

You are not permitted to use your own graphing calculator unless your teacher has cleared the memory immediately prior to this examination. The only graphing calculators permitted are TI-82, TI-83, TI-83 Plus, TI-84, or TI-84 Plus.

If the question indicates that you are not to use a graphing calculator, you are still permitted to use a calculator to perform arithmetic operations.

Calculators are not to be shared.

Graph paper, scrap paper, and formula sheets are provided at the end of this booklet. These pages can be removed from the booklet for your use during the examination.

Note: Diagrams are not necessarily drawn to scale.

In this part of the examination, there are 25 selected-response questions*, each with a value of 1 point. Read each question carefully, and decide which of the responses best answers the question being asked.

You are provided a separate student answer sheet. In the selected-response section of the student answer sheet, fill in the bubble that corresponds to your choice as shown in the example below. Use an HB pencil only.

Example
1.   What are the roots of $x^2 + 3x - 4 = 0$?

    A. 4 and 1   B.   −4 and 3
    C. −4 and 1   D.   4 and 3

    (On student answer sheet)

    1.   Ⓐ Ⓑ ⬤ Ⓓ

If you wish to change an answer, please ensure that you erase your first answer completely on the student answer sheet. Calculations or rough work on the selected-response pages of the examination booklet will not be scored.

*Note: As of January 2008 there are 35 selected response questions on the NSE Math exams.

1. The function $t_n = 2n^2 + 3n - 1$ is used to generate a quadratic sequence. What is the value of D_2?
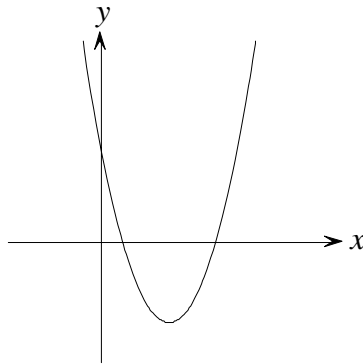
   A. 1

   B. 2

   C. $-\frac{3}{4}$

   D. 4

2. What is the mapping rule that maps $y = x^2$ onto $-\frac{1}{2}(y - 5) = (x + 7)^2$?

   A. $(x, y) \rightarrow (x + 7, -\frac{1}{2}y - 5)$

   B. $(x, y) \rightarrow (x - 7, -\frac{1}{2}y - 5)$

   C. $(x, y) \rightarrow (x + 7, -2y - 5)$

   D. $(x, y) \rightarrow (x - 7, -2y + 5)$

3. What are the coordinates of the vertex of the quadratic function having a maximum value of 20 and x-intercepts located at (5, 0) and $(-1, 0)$?

   A. (3, 20)

   B. (2, 0)

   C. (2, 20)

   D. (3, 0)

4. The graph of a quadratic function $y = ax^2 + bx + c$ is given below.

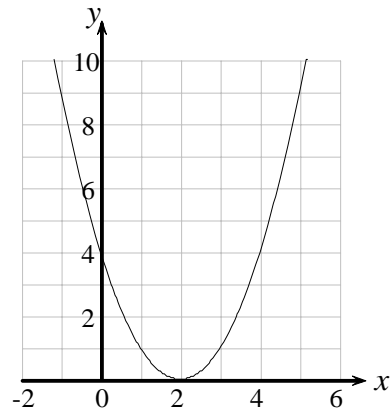   When solving for x in the equation $ax^2 + bx + c = 0$, the solution(s) represents

   A. the x coordinate of the vertex of the quadratic function $y = ax^2 + bx + c$

   B. the maximum or minimum value of x

   C. the x-intercepts of $y = ax^2 + bx + c$

   D. the x coordinate of the y-intercept of the quadratic function $y = ax^2 + bx + c$

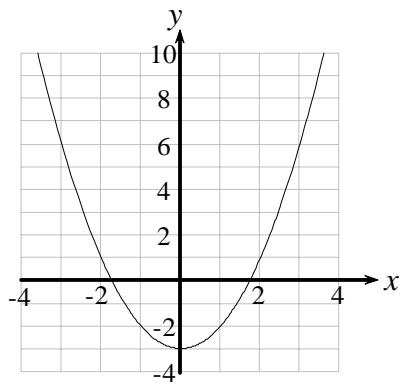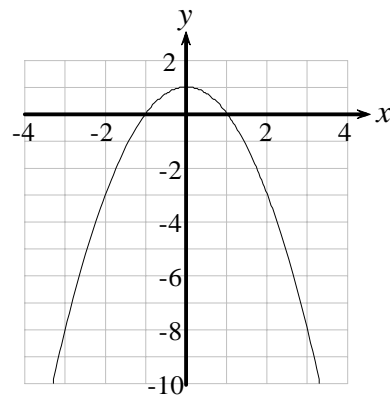5. In which of the following quadratics is the discriminant equal to zero?

A.
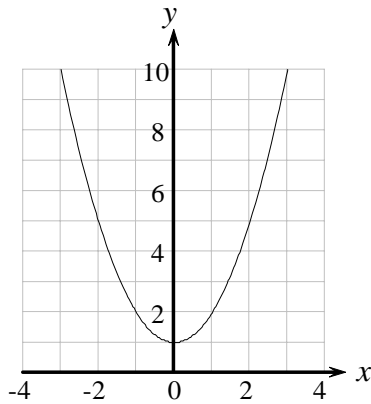
B.

C.

D.

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6. The vertex of the parabola represented by $y = 2x^2 - 4x - 4$ is

A. $(1, -6)$ B. $(1, -5)$
C. $(-1,\ 2)$ D. $(-1,\ 6)$

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7. The axis of symmetry for the parabola defined by $y = x^2 - 6x + 12$ is

A. $x = -4$ B. $x = -6$
C. $x = 3$ D. $x = 12$

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8. A cannon ball is shot into the air and its height in metres is represented by the equation
$h = 1.5 + 23.1t - 4.9t^2$ where t is time in seconds. How high does the cannon ball go?
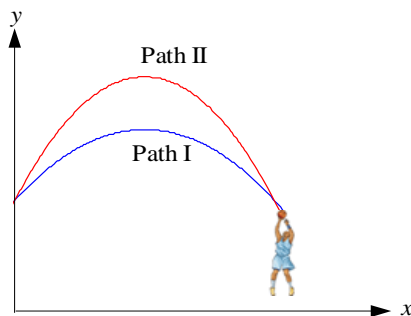
A. 23.10 m   B. 4.78 m
C. 28.73 m   D. 2.36 m

9. A basketball player is trying to increase her shot accuracy. She stays in the same position on the court and increases the arc of the flight path of the ball.

Her coach graphs the quadratic function $\frac{1}{a}(y-k)=(x-h)^2$ to model parabolic path I of the basketball. The coach then changes certain values in the given equation to graph path II. Which value(s) did the coach NOT change ?

A. a
B. h
C. k
D. h and a

10. Which one of the following tables of values was generated by an exponential function?

A.

x	-1	2	3	5
y	$\frac{1}{2}$	1	2	4

B.

x	-1	1	3	5
y	1	3	9	27

C.

x	-1	0	1	2
y	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$

D.

x	1	2	3	4
y	1	4	9	16

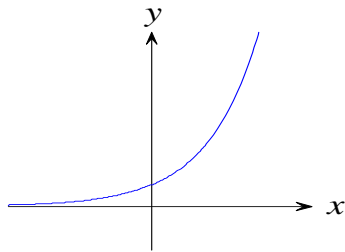11. The value of $\left(6^0+3^2\right)^{-3}$ is:

A. $\frac{1}{1000}$
B. $\frac{1}{10}$

C. -1000
D. $\frac{1}{729}$

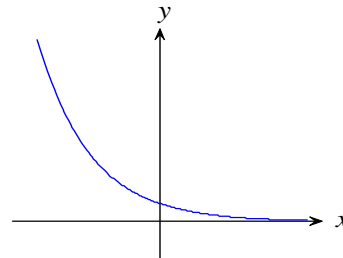12. Which of the following is equal to $b^{-\frac{2}{3}}$ if $b \neq 0$?

A. $-\sqrt[3]{b^2}$
B. $\frac{1}{\sqrt[3]{b^2}}$

C. $\sqrt{b^3}$
D. $\frac{1}{\sqrt{b^3}}$

13. Which graph represents $y = a \cdot b^x$, where $a > 0$ and $0 < b < 1$?
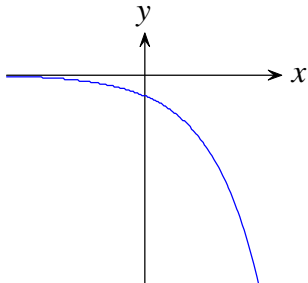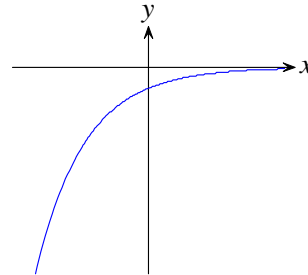
A.

B.

C.

D.

14. A particular bacteria population doubles every 7 days. Which function determines the number of bacteria N after t days, given the initial amount of 500 bacteria?

A. $N = 500(7)^{\frac{t}{2}}$

B. $N = 500(2)^{7t}$

C. $N = 500(7)^{2t}$

D. $N = 500(2)^{\frac{t}{7}}$

15. What could be the next correct step in solving this equation $(2^{x+5})(2^{2x}) = 12$?

A. $4^{3x+5} = 12$

B. $2^{3x+5} = 12$

C. $4^{2x^2+10x} = 12$

D. $\log_2(3x + 5) = \log_2 12$

16. $\log_t r = s$ can also be expressed as

A. $s^t = r$

B. $t^r = s$

C. $r^s = t$

D. $t^s = r$

17. The value of x in the equation $\log_5 x = 0$ is
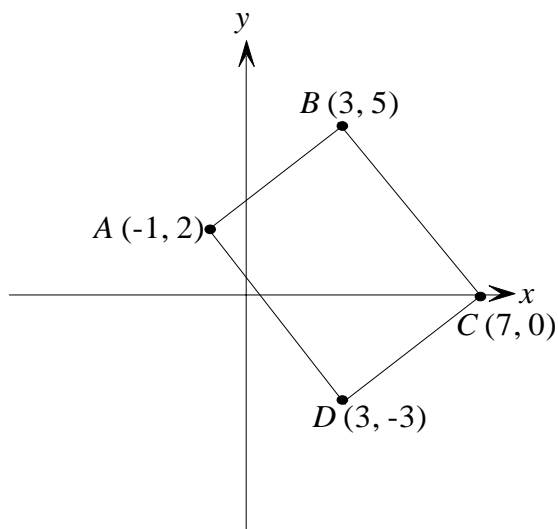
A. 0

B. $\frac{1}{5}$

C. 1

D. 5

18. The expression $\dfrac{\log_2 16}{\log_2\left(\frac{1}{2}\right)}$ is equal to
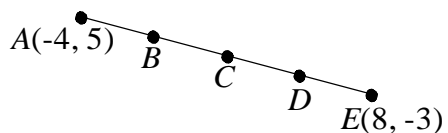
A. −4

B. 3

C. 5

D. 32

19. Given the following figure:

Which of the following statements is **false**?

A. $AB = DC$

B. The midpoint of $\overline{AD}$ is $\left(1, -\frac{1}{2}\right)$.

C. The diagonals $\overline{AC}$ and $\overline{BD}$ are of equal lengths.

D. The diagonals $\overline{AC}$ and $\overline{BD}$ bisect each other.

20. The segment below is to be cut into 4 equal pieces.


What are the coordinates of B?

A. $(2,1)$

B. $(-1, 2)$

C. $\left(1, \frac{1}{2}\right)$

D. $(-1, 3)$

21. What is the converse of this statement?
If two chords are equidistant from the centre of a circle, then the chords are congruent.

A. If two chords are equidistant from the centre of a circle, then they are parallel.
B. Two chords that are the same distance from the centre of a circle are congruent.
C. If two chords of a circle are congruent, then they are equidistant from the centre of the circle.
D. If two chords of a circle are congruent, then they pass through the centre of the circle.

22. Consider the following Venn diagram.

| Event K | Event L |
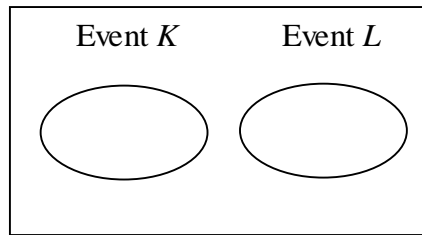
Which of the following is correct?

A. P(K or L) = P(K) + P(L)   B. P(K and L) = P(K) + P(L)
C. P(K or L) = P(K) x P(L)   D. P(K and L) = P(K) x P(L)

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23. Your math teacher gives your class a list of eight questions to study. Five of the eight questions will be randomly selected for the next test. If you study only the first five questions from the list, the probability that all of those five questions will be on the test is

A. $\dfrac{1}{_8C_5}$ B. $\dfrac{1}{_8P_5}$

C. $\frac{5}{8}$ D. $\frac{1}{8}$

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24. The value of $\frac{500!}{499!}$ is

A. 1   B. 1.002
C. 500   D. undefined

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25. A bag contains 9 marbles, 4 of which are red. What expression represents the probability of selecting three red marbles when three marbles are drawn at random?

A. $\dfrac{1}{_9C_3}$ B. $\dfrac{4}{_9C_3}$

C. $\dfrac{4}{_9P_3}$ D. $\dfrac{1}{_9P_3}$

Constructed-Response Questions
(Total Value: 75 points)

Read each question carefully, and be sure to write your response in the box and space provided. If the answer box indicates that you are to write your response in the box and space provided. If the answer box indicates that you are to show your work, then points will be awarded for your correct work and your correct final answer. The method used to solve a problem must clearly be shown even when using a graphing calculator. If the answer box requires that just a final answer be provided, then points will be awarded for the correct answer only.

When working with decimal values, you may round off to the hundredths place in your final answer only. If any decimal values are rounded prior to the final step of the solution, at least 4 decimal places must be kept.

With the exception of the probability unit, all answers must be given in simplified form.

26. (a) Write the function, in transformational form, that represents the following parabola.

(3 points)
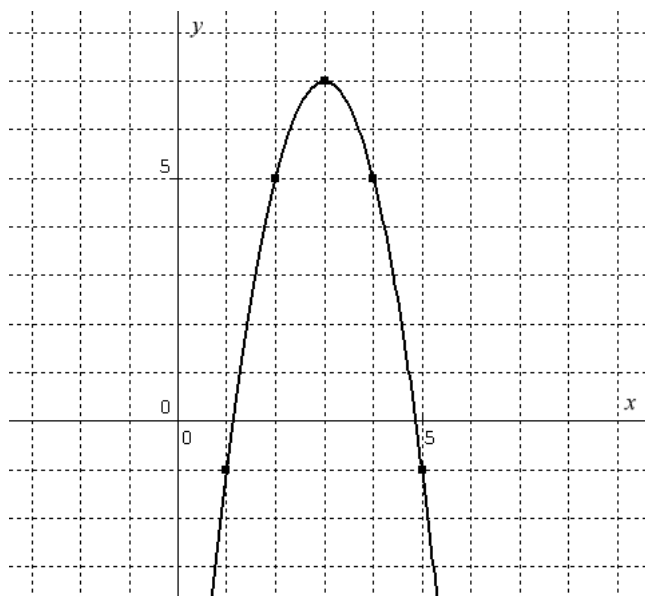
Show your work above and write your conclusion or final answer in the box below.

(b) State the domain and range of the function.

(2 points)

Domain: | Final Answer |

Range: | Final Answer |

27. Calculate the discriminant of $3x^2 + 8x + 8 = 0$ and explain what the result tells you about the graph of $y = 3x^2 + 8x + 8$. (2 points)

Show your work above and write your conclusion or final answer in the box below.

28. The severity of an automobile crash increases significantly as the speed increases. The table shows the relationship between the speed and a crash severity index.

Speed (km/h)	10	20	30	40	50
Crash severity index	1.20	4.40	9.60	16.80	26.00

(a) Jimmy claims that a quadratic function would best model this situation. Is Jimmy's claim correct? Explain. (2 points)

(b) What speed would have a crash severity index of 50.40? (3 points)

Show your work above and write your conclusion or final answer in the box below.

29. Solve for x in each of the following equations using a different algebraic method for each.

(a) $x^2 = x + 6$ (3 points)

Show your work above and write your conclusion or final answer in the box below.

(b) $2x^2 + 7x - 4 = 0$ (3 points)

Show your work above and write your conclusion or final answer in the box below.

30. A football is kicked into the air. The equation $h = -4.9t^2 + 9.8t + 1$ expresses the relationship between height h in metres and time t in seconds.

(a) Determine the maximum height reached by the football. (3 points)

Show your work above and write your conclusion or final answer in the box below.

(b) At what time(s) after the kick is the ball at a height of 5 m? (3 points)

Show your work above and write your conclusion or final answer in the box below.

31. A golf ball is hit from ground level in a flat field and reaches a maximum height of 25 m. The ball first hits the ground 100 m away while following a parabolic path.

(a) Draw a diagram and include all important information needed to model this problem. Remember to label your axes. (2 points)

(b) How high is the golf ball above the ground when it is at a horizontal distance of 20 m from where it was hit? (4 points)

Show your work above and write your conclusion or final answer in the box below.

32. Solve for x in each of the three parts below. At least two of the three parts must be solved algebraically.

(a) $9^{x+2} = 27^{3x+1}$

(3 points)

> **Show your work above and write your conclusion or final answer in the box below.**

(b) $2^{3x-1} = 17$

(3 points)

> **Show your work above and write your conclusion or final answer in the box below.**

(c) $\log_5(2x) + \log_5 10 = \log_2 16$

(3 points)

> **Show your work above and write your conclusion or final answer in the box below.**

33. Given $y = 0.5(2)^x + 4$.

(a) Determine the coordinates of the y-intercept. (1 point)

Final Answer	

(b) Write the equation of the horizontal asymptote. (1 point)

Final Answer	

(c) Indicate whether the function above represents a growth curve or a decay curve. Explain how you know. (2 points)

34. Express the following expression as a single logarithm. Simplify your answer. (3 points)

$\log 2x + 3(\log y - \log z)$

Show your work above and write your conclusion or final answer in the box below.

35. (a) Evaluate: (2 points)

    (i) $\log_2 8 = $ _____

    (ii) $- \log_{\frac{1}{2}} 8 = $ _____

  (b) Evaluate: (2 points)

    (i) $\log_3 9 = $ _____

    (ii) $- \log_{\frac{1}{3}} 9 = $ _____

  (c) Based on the answers obtained in parts (a) and (b), write an expression equivalent
to $-\log_{\frac{1}{b}} N$. (1 point)

Final Answer	

36. Marla tries to evaluate 0^{-3} (zero to the exponent negative 3) on her calculator and gets an error message. Explain why she got an error message. (2 points)

37. Describe a situation that could be modelled by the function $P = 1000(2)^{\frac{t}{5}}$. (3 points)

38. A circle has a diameter of 26 cm. A chord in the same circle measures 24 cm. What is the shortest distance from the centre of the circle to the chord? (3 points)

Show your work above and write your conclusion or final answer in the box below.

39. The coordinates of the corners of a stained glass window are $A(-1, -1)$, $B(3, 2)$, $C(6, -2)$ and $D(2, -5)$.

(a) Show algebraically that the diagonals have the same length. (2 points)

(b) Provide calculations to determine whether or not the diagonals of the window bisect each other. (3 points)

Show your work above and write your conclusion or final answer in the box below.

Mathematics 12

40. Write any true statement for which the converse is NOT true. (1 point)

41. There are 15 jellybeans randomly distributed in a jar; 5 are yellow and 10 are orange. You reach into the jar and, without looking, remove 2 jellybeans. What is the probability that you will remove 2 yellow jellybeans? (3 points)

Show your work above and write your conclusion or final answer in the box below.

42. In a group of 15 people, 4 are left-handed and 11 are right-handed. Seven people are selected at random from this group.

(a) What is the probability that everyone selected is right-handed? (2 points)

Show your work above and write your conclusion or final answer in the box below.

(b) If Sarah and Mike, two of the left handers, have already been chosen, what is the probability that all the others members selected will be right-handed? (2 points)

Show your work above and write your conclusion or final answer in the box below.

43. Create a real-life problem that demonstrates P(A or B) = P(A) + P(B) − P(A and B) when the events A and B are NOT mutually exclusive.
(You don't have to solve the problem.) (3 points)

You have reached the end of the SAMPLE Mathematics 12 Examination.
Please check your work to ensure you have completed all questions.

Formula Sheet – Mathematics 12

Quadratics Unit

General form: $y = ax^2 + bx + c$

Standard form: $y = a(x-h)^2 + k$

Transformational form: $\dfrac{1}{a}(y-k) = (x-h)^2$

If $ax^2 + bx + c = 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Exponential Growth Unit

$y = ab^x$

$\log_a(xy) = \log_a x + \log_a y$

$\log_a(x \div y) = \log_a x - \log_a y$ or $\log_a\left(\dfrac{x}{y}\right) = \log_a x - \log_a y$

$\log_a x^b = b(\log_a x)$

Circle Geometry Unit

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The coordinates of M are: $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2} \right)$

$$m = \frac{\Delta y}{\Delta x}$$

Probability Unit

$$P(A \text{ and } B) = P(A) \times P(B)$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$_nP_r = \frac{n!}{(n-r)!} \qquad\qquad _nC_r = \frac{n!}{r!(n-r)!}$$

Selected Response Answers

1.	D	14.	D
2.	D	15.	B
3.	C	16.	D
4.	C	17.	C
5.	B	18.	A
6.	A	19.	C
7.	C	20.	D
8.	C	21.	C
9.	B	22.	A
10.	B	23.	A
11.	A	24.	C
12.	B	25.	B
13.	B		

Question 26(a)　　　　　　　　　　　　　　　　　　　　(3 points)

0.5 pt　**0.5 pt**

$$\frac{1}{a}(y-7)=(x-3)^2$$

$$\frac{1}{a}(5-7)=(2-3)^2 \quad \text{1 pt}$$

$$\frac{-2}{a}=1$$

$$a=-2 \quad \text{0.5 pt}$$

$$\therefore \ -\frac{1}{2}(y-7)=(x-3)^2 \quad \text{0.5 pt}$$

To go from (7, 3) to (4, 5) there is a horizontal translation is 1 and a vertical translation of -2. If there was no vertical stretch the vertical translation should be 1, therefore the vertical stretch must be –2.

1.5 pt

$$\therefore \ -\frac{1}{2}(y-7)=(x-3)^2 \quad \text{1.5 pt}$$

OR

QuadReg on TI-83

$$y = ax^2 + bx + c \quad \text{1 pt}$$
$$a = -2$$
$$b = 12$$
$$c = -11$$

$$y = -2x^2 + 12x - 11$$
$$y + 11 = -2x^2 + 12x$$
$$y + 11 = -2(x^2 - 6x) \quad \text{0.5 pt}$$
$$y + 11 = -2(x^2 - 6x + 9) + 18 \quad \text{1 pt}$$
$$y - 7 = -2(x-3)^2$$
$$-\frac{1}{2}(y-7) = (x-3)^2 \quad \text{0.5 pt}$$

Question 26(b)　　　　　　　　　　　　　　　　　　　　(3 points)

Domain: $\{x \in \mathbb{R}\}$ or $(-\infty, \infty)$　**1 pt**

Range: $\{y \mid y \leq 7, y \in \mathbb{R}\}$ or $(-\infty, 7]$　**1 pt**

Question 27 (2 points)

$D = b^2 - 4ac$

$= (8)^2 - 4(3)(8)$ [0.5 pt]

$= 64 - 96$

$= -32$ [0.5 pt]

[0.5 pt] Since the discriminant is less than zero, the graph will have no *x*-intercepts. [0.5 pt]

Question 28 (a) (2 points)

Speed (km/h)	10	20	30	40	50
Crash severity index	1.20	4.40	9.60	16.80	26.00

3.2   5.2   7.2   9.2

2   2   2

[1 pt] Yes Jimmy is correct because the second-level differences, D_2, are constant. [1 pt]

OR

[1 pt] Yes Jimmy is correct because the QuadReg on the TI-83 shows an R2 value of 1. [1 pt]

QuadReg TI-83

$y = ax2 + bx + c$

$a = 0.01$
$b = 0.02$
$c = 0$
$R^2 = 1$

$y = ax^2 + bx + c$ $c = 0$ (obtained from table)   [0.5 pt]

$\therefore y = 0.01x^2 + 0.02x$

$0.01x^2 + 0.02x = 50.4$ [0.5 pt]

$0.01x^2 + 0.02x - 50.4 = 0$

$2a(\Delta x)^2 = 2$ $(10, 1.2) \Rightarrow$ $y = 0.01x^2 + bx$

$2a(100) = 2$ $1.2 = 0.01(10)^2 + 10b$ solve using any method to get: [0.5 pt]

$200a = 2$ $1.2 = 1 + 10b$ $x = 70$ [0.5 pt]

$a = 0.01$ [0.5 pt] $0.2 = 10b$

$0.02 = b$ [0.5 pt]

OR

Using QuadReg on the TI-83:   [0.5 pt]

$y = 0.01x^2 + 0.02x$ [1 pt]

Table of values on TI-83   [0.5 pt]

$0.01x^2 + 0.02x = 50.4$ [0.5 pt]

$0.01x^2 + 0.02x - 50.4 = 0$

x	y
60	37.2
70	50.4
80	65.6

[1 pt]

OR

solve using any method to get: [0.5 pt]

$x = 70$ [0.5 pt]

OR

Speed (km/h)	10	20	30	40	50	**60**	**70**
Crash severity index	1.20	4.40	9.60	16.80	26.00	**37.20**	**50.40**

[1 pt]

3.2   5.2   7.2   9.2   **11.2**   **13.2**   [1 pt]
for 1st level differences

2   2   2   **2**   **2**

$x = 70$ [1 pt]

Question 29 (a)　　　　　　　　　　　　　　　　　　　　　(3 points)

$$x = \frac{1 \pm \sqrt{(-1)^2 - (4)(1)(-6)}}{2(1)}$$　　1 pt

$$= \frac{1 \pm \sqrt{25}}{2}$$　　1 pt

$$x = \frac{1+5}{2}$$　　　$$x = \frac{1-5}{2}$$

0.5 pt　　$$= 3$$　　　　$$= -2$$　　0.5 pt

$$x^2 - x - 6 = 0$$

$$(x-3)(x+2) = 0$$　　1 pt

0.5 pt　　$$x - 3 = 0$$　　$$x + 2 = 0$$　　0.5 pt

0.5 pt　　　　$$x = 3$$　　　$$x = -2$$　　0.5 pt

OR

$$x^2 - x = 6$$

$$x^2 - x + \frac{1}{4} = 6 + \frac{1}{4}$$　　1 pt

$$\left(x - \frac{1}{2}\right)^2 = \frac{25}{4}$$　　0.5 pt

$$\sqrt{\left(x - \frac{1}{2}\right)^2} = \sqrt{\frac{25}{4}}$$

$$x - \frac{1}{2} = \pm\frac{5}{2}$$　　0.5 pt

$$x = \pm\frac{5}{2} + \frac{1}{2}$$

$$x = \frac{5}{2} + \frac{1}{2}$$　　　$$x = -\frac{5}{2} + \frac{1}{2}$$

0.5 pt　　$$= 3$$　　　　$$= -2$$　　0.5 pt

Question 29 (b) (3 points)

$$x = \frac{-7 \pm \sqrt{(7)^2 - (4)(2)(-4)}}{2(2)} \quad \boxed{\text{1 pt}}$$

$$= \frac{-7 \pm \sqrt{81}}{4} \quad \boxed{\text{1 pt}}$$

$$x = \frac{-7 - 9}{4} \qquad x = \frac{-7 + 9}{4}$$

$$\boxed{\text{0.5 pt}} = -4 \qquad\qquad = \frac{1}{2} \quad \boxed{\text{0.5 pt}}$$

$$2x^2 + 8x - x - 4 = 0$$

$$2x(x+4) - 1(x+4) = 0$$

$$(2x-1)(x+4) = 0 \quad \boxed{\text{1 pt}}$$

$$\boxed{\text{0.5 pt}} \; 2x - 1 = 0 \qquad x + 4 = 0 \quad \boxed{\text{0.5 pt}}$$

$$2x = 1 \qquad\qquad x = -4 \quad \boxed{\text{0.5 pt}}$$

$$\boxed{\text{0.5 pt}} \qquad x = \frac{1}{2}$$

OR

$$2x^2 + 7x = 4$$

$$x^2 + \frac{7}{2}x = 2 \quad \boxed{\text{1 pt}}$$

$$x^2 + \frac{7}{2}x + \frac{49}{16} = 2 + \frac{49}{16} \quad \boxed{\text{0.5 pt}}$$

$$\left(x + \frac{7}{4}\right)^2 = \frac{81}{16}$$

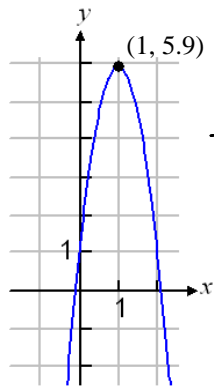$$\sqrt{\left(x + \frac{7}{4}\right)^2} = \sqrt{\frac{81}{16}} \quad \boxed{\text{0.5 pt}}$$

$$x + \frac{7}{4} = \pm \frac{9}{4}$$

$$x = \pm \frac{9}{4} - \frac{7}{4}$$

$$x = \frac{9}{4} - \frac{7}{4} \qquad x = -\frac{9}{4} - \frac{7}{4}$$

$$= \frac{2}{4} \qquad\qquad = -\frac{16}{4}$$

$$= -4 \quad \boxed{\text{0.5 pt}}$$

$$\boxed{\text{0.5 pt}} \qquad = \frac{1}{2}$$

Question 30 (a)   (3 points)

$h = -4.9t^2 + 9.8t + 1$

$\quad = -4.9(t^2 - 2t) + 1$ — 0.5 pt

$\quad = -4.9(t^2 - 2t + 1) + 1 + 4.9$ — 1 pt

$\quad = -4.9(t - 1)^2 + 5.9$ — 0.5 pt

— 1 pt

The maximum height is 5.9 metres.

$h = -4.9t^2 + 9.8t + 1$

$h - 1 = -4.9(t^2 - 2t)$ — 1 pt

$h - 1 - 4.9 = -4.9(t^2 - 2t + 1)$ — 0.5 pt

$h - 5.9 = -4.9(t - 1)^2$ — 0.5 pt

$-\dfrac{1}{4.9}(h - 5.9) = (t - 1)^2$

— 1 pt

The maximum height is 5.9 metres.

OR

$t = -\dfrac{b}{2a}$

$\quad = -\dfrac{9.8}{2(-4.9)}$ — 1 pt

$\quad = 1$

$h = -4.9(1)^2 + 9.8(1) + 1$

$\quad = 5.9$ — 1 pt

— 1 pt

The maximum height is 5.9 metres.

$h = \dfrac{4ac - b^2}{4a}$

$\quad = \dfrac{4(-4.9)(1) - (9.8)^2}{4(-4.9)}$ — 2 pts

$\quad = 5.9$

The maximum height is 5.9 metres.

— 1 pt

OR

$y = -4.9x^2 + 9.8x + 1$

(1, 5.9)

— 2 pts

The maximum height is 5.9 metres.

— 1 pt

Question 30(b)   (3 points)

$$-4.9t^2 + 9.8t + 1 = 5$$ `1 pt`
$$-4.9t^2 + 9.8t - 4 = 0$$

Left side:

$$t = \frac{-9.8 \pm \sqrt{(9.8)^2 - (4)(-4.9)(-4)}}{2(-4.9)}$$ `1 pt`

$$= \frac{-9.8 \pm \sqrt{17.64}}{-9.8}$$

OR

$$t = \frac{-9.8 + 4.2}{-9.8} \qquad t = \frac{-9.8 - 4.2}{-9.8}$$

$$t \doteq 0.57 \qquad \qquad \doteq 1.43$$

`0.5 pt`   `0.5 pt`

Right side:

$$-4.9t^2 + 9.8t = 4$$
$$t^2 - 2t \doteq -0.8163$$
$$t^2 - 2t + 1 \doteq 0.1837$$ `0.5 pt`
$$(t-1)^2 \doteq 0.1837$$
$$\sqrt{(t-1)^2} \doteq \sqrt{0.1837}$$
$$t - 1 \doteq \pm 0.4286$$ `0.5 pt`

$$t \doteq -0.4286 + 1 \qquad t \doteq 0.4286 + 1$$
$$t \doteq 0.57 \text{ `0.5 pt`} \qquad t \doteq 1.43 \text{ `0.5 pt`}$$

The ball will be at 5 m at 0.57 s and at 1.43 s.

OR

$$y = -4.9t^2 + 9.8t - 4$$ `1 pt`

(0, 0.57)   (0, 1.43)

`1 pt`

The ball will be at 5 m at 0.57 s and at 1.43 s. `0.5 pt`

`0.5 pt`

$$y_1 = -4.9t^2 + 9.8t + 1$$ `1 pt`
$$y_2 = 5$$

(5, 0.57)   (5, 1.43)

`1 pt`

The ball will be at 5 m at 0.57 s and at 1.43 s. `0.5 pt`

`0.5 pt`

Question 31   (4 points)

(50, 25)

(0, 0)

(100, 0)

0.5 pt for each of the following:
- ✓ the graph has a parabolic form, opens downward and axes are labeled.
- ✓ Passes through point (0, 0) and point is labeled.
- ✓ Passes through (100, 0) and point is labeled.
- ✓ Vertex identified and labeled at (50, 25)

Question 31(b)   (4 points)

$$y = a(x - 50)^2 + 25 \quad \boxed{1 \text{ pt}}$$

$$(0, \, 0) \Rightarrow 0 = a(0 - 50)^2 + 25 \quad \boxed{0.5 \text{ pt}}$$

$$-25 = 2500a$$

$$-0.01 = a \quad \boxed{0.5 \text{ pt}}$$

$$y = -0.01(x - 50)^2 + 25$$

$$= -0.01(20 - 50)^2 + 25 \boxed{1 \text{ pt}}$$

$$= 16 \quad \boxed{1 \text{ pt}}$$

The ball will be at a height of 16 m at a horizontal distance of 20 m.

Question 32 (a)   (3 points)

$$\left(3^2\right)^{x+2} = \left(3^3\right)^{3x+1}$$ `1 pt`

$$3^{2x+4} = 3^{9x+3}$$ `0.5 pt`

$$\therefore 2x+4 = 9x+3$$ `1 pt`

$$-7x = -1$$

$$x = \frac{1}{7}$$ `0.5 pt`

$$(x+2)\log 9 = (3x+1)\log 27$$ `1 pt`

$$x+2 = \frac{(3x+1)\log 27}{\log 9}$$ `1 pt`

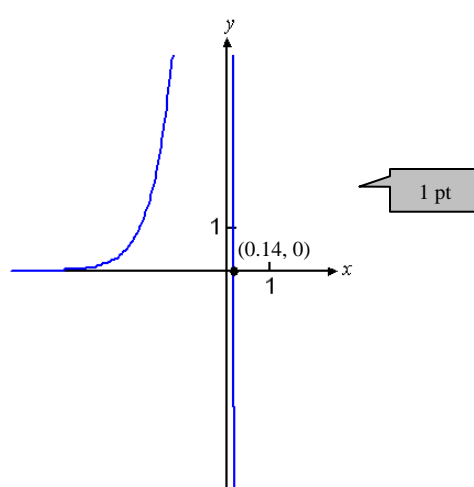$$x+2 = (3x+1)(1.5)$$
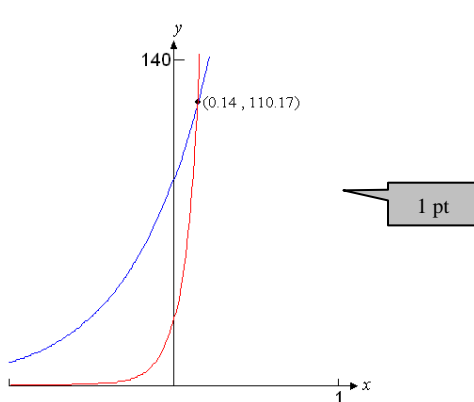
$$x+2 = 4.5x+1.5$$

$$-3.5x = -0.5$$

$$x = \frac{1}{7}$$ `1 pt`

OR

$$y_1 = 9^{x+2}$$
$$y_2 = 27^{3x+1}$$ `1 pt`

(0.14 , 110.17)

`1 pt`

$$\therefore x = 0.14$$ `1 pt`

$$y = 9^{x+2} - 27^{3x+1}$$ `1 pt`

(0.14, 0)

`1 pt`

$$\therefore x = 0.14$$ `1 pt`

Question 32 (b) (3 points)

$\log 2^{3x-1} = \log 17$ 1 pt

$3x - 1 = \dfrac{\log 17}{\log 2}$ 0.5 pt

$3x - 1 \doteq 4.0875$ 0.5 pt

$3x \doteq 5.0875$

$x \doteq 1.70$ 1 pt

$y = 2^{3x-1} - 17$ 1 pt

OR

$y_1 = 2^{3x-1}$
$y_2 = 17$ 1 pt

(1.70, 17)

1 pt

$\therefore x = 1.70$ 1 pt

1 pt

(1.70, 0)
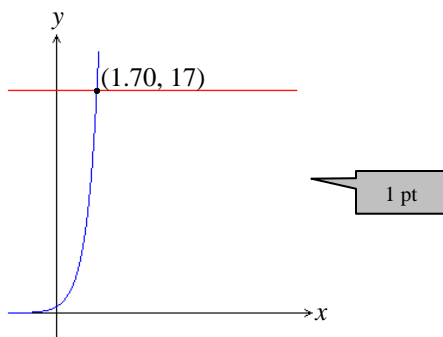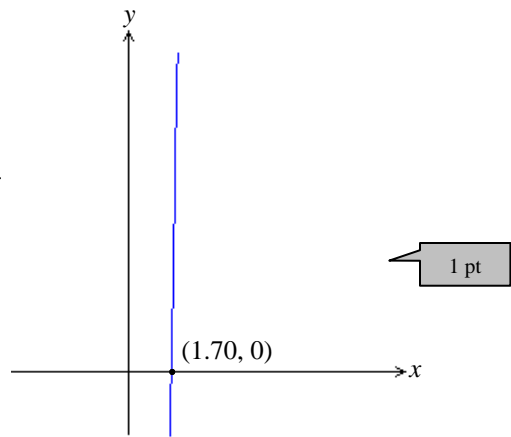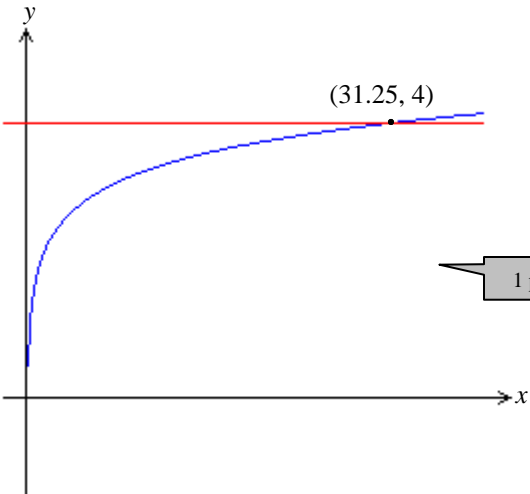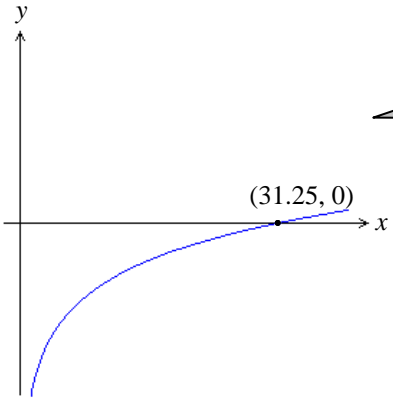
$\therefore x = 1.70$ 1 pt

Question 32 (c)   (3 points)

$$\log_5\left(2x\cdot10\right)=4$$ 1 pt

$$\log_5 20x = 4$$ 0.5 pt

$$20x = 5^4$$ 1 pt

$$20x = 625$$

$$x = 31.25$$ 0.5 pt

$$y = \log_5 2x + \log_5 10 - 4$$ 1 pt

OR

$$y_1 = \log_5 2x + \log_5 10$$ 1 pt

$$y_2 = 4$$

1 pt

(31.25, 0)

(31.25, 4)

1 pt

$$\therefore x = 31.25$$ 1 pt

$$\therefore x = 31.25$$ 1 pt

Question 33 (a) (1 point)
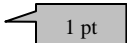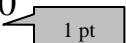
$(0,\ 4.5)$ `1 pt`

Question 33 (b) (1 point)

$y = 4$ `1 pt`

Question 33 (c) (2 points)

It is a growth curve. `1 pt`

The function is of the form $y = ab^x + c$, which is a growth curve if a > 0 and b > 1. `1 pt`

Question 34 (3 points)

$$\log 2x + 3\left(\log \frac{y}{z}\right)$$ — 1 pt

$$\log 2x + \log\left(\frac{y^3}{z^3}\right)$$ — 1 pt

$$\log\left(\frac{2xy^3}{z^3}\right)$$ — 1 pt

OR

$$\log 2x + 3\log y - 3\log z$$ — 0.5 pt

$$\log 2x + \log y^3 - \log z^3$$ — 0.5 pt

$$\log 2xy^3 - \log z^3$$ — 1 pt

$$\log\left(\frac{2xy^3}{z^3}\right)$$ — 1 pt

Question 35(a) (2 points)

(i) 3 — 1 pt   (ii) 3 — 1 pt

Question 35(b) (2 points)

(i) 2 — 1 pt   (ii) 2 — 1 pt

Question 35(c) (1 point)

$$-\log_{\frac{1}{b}} N = \log_b N$$ — 1 pt

Question 36 (2 points)

$$0^{-3} = \frac{1}{0^3} = \frac{1}{0}$$ 1 pt

Division by 0 is undefined. 1 pt

Question 37 (3 points)

✓ Initial amount of 1000 1 pt

✓ Doubles 1 pt

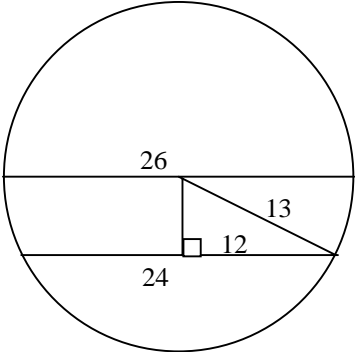✓ Doubling period is 5 units of time 1 pt

Examples:

A $1000 investment doubles every 5 years.

The number of bacteria starts at 1000 and doubles every 5 hours.
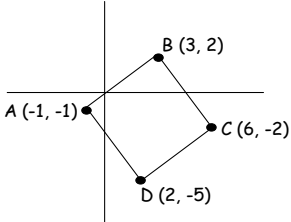
Question 38 (3 points)

$$h^2 = x^2 + y^2$$
$$13^2 = 12^2 + y^2 \quad \boxed{1 \text{ pt}}$$
$$169 = 144 + y^2 \quad \boxed{0.5 \text{ pt}}$$
$$25 = y^2 \quad \boxed{0.5 \text{ pt}}$$
$$5 = y \quad \boxed{1 \text{ pt}}$$

The chord is 5 cm from the centre.

Question 39(a) (2 points)

$$D_{\overline{AC}} = \sqrt{\left(6-(-1)\right)^2 + \left(-2-(-1)\right)^2} \quad \boxed{0.5 \text{ pt}}$$
$$= \sqrt{49+1}$$
$$= \sqrt{50} \quad \boxed{0.5 \text{ pt}}$$
$$= 5\sqrt{2}$$

$$D_{\overline{BD}} = \sqrt{\left(3-2\right)^2 + \left(2-(-5)\right)^2} \quad \boxed{0.5 \text{ pt}}$$
$$= \sqrt{1+49}$$
$$= \sqrt{50} \quad \boxed{0.5 \text{ pt}}$$
$$= 5\sqrt{2}$$

Question 39(b) (3 points)

$$\text{Midpoint of } \overline{AC} = \left(\frac{-1+6}{2}, \frac{-1+(-2)}{2}\right)$$
$$= \left(\frac{5}{2}, \frac{-3}{2}\right) \quad \boxed{1 \text{ pt}}$$

$$\text{Midpoint of } \overline{BD} = \left(\frac{3+2}{2}, \frac{2+(-5)}{2}\right)$$
$$= \left(\frac{5}{2}, \frac{-3}{2}\right) \quad \boxed{1 \text{ pt}}$$

Since the diagonals have the same midpoint, they bisect each other. $\boxed{1 \text{ pt}}$

Question 40 (1 point)

Some examples:

- ✓ If a quadrilateral is a rhombus, then the diagonals are perpendicular.

- ✓ If a quadrilateral is a square, the diagonals are congruent.

- ✓ If you have a million dollars, then you are rich.

Question 41 (3 points)

1 pt

$$\frac{_5C_2}{_{15}C_2} = \frac{10}{105} = \frac{2}{21}$$

1 pt

1 pt

OR

1 pt

$$\frac{5\times4}{15\times14} = \frac{20}{210} = \frac{2}{21}$$

1 pt

1 pt

Question 42(a) (2 points)

0.5 pt

$$\frac{_{11}C_7}{_{15}C_7} = \frac{330}{6435} = \frac{2}{39}$$

1 pt

0.5 pt

OR

0.5 pt for numerators

$$\frac{11}{15} \times \frac{10}{14} \times \frac{9}{13} \times \frac{8}{12} \times \frac{7}{11} \times \frac{6}{10} \times \frac{5}{9} = \frac{2}{39}$$

0.5 pt

1 pt

for denominators

Question 42(b) (2 points)

0.5 pt

$$\frac{_{11}C_5}{_{13}C_5} = \frac{462}{1287} = \frac{14}{39}$$

1 pt

0.5 pt

OR

0.5 pt for numerators

$$\frac{11}{13} \times \frac{10}{12} \times \frac{9}{11} \times \frac{8}{10} \times \frac{7}{9} = \frac{14}{39}$$

1 pt

0.5 pt

for denominators

Question 43 (3 points)

Examples:
- ✓ What is the probability of being 6'3" or left handed?

- ✓ What is the probability of becoming a teacher or a mother?

- ✓ What is the probability of being over 30 and having O positive blood?